Mathematics: applications and interpretation
Standard level
Paper 2

Tuesday 1 November 2022 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 17]

Elsie, a librarian, wants to investigate the length of time, T minutes, that people spent in her library on a particular day.

(a) State whether the variable T is discrete or continuous. [1]

Elsie's data for 160 people who visited the library on that particular day is shown in the following table.

T (minutes)	$0 \le T < 20$	$20 \le T < 40$	$40 \le T < 60$	$60 \le T < 80$	$80 \le T < 100$
Frequency	50	62	k	14	8

(b) Find the value of k. [2]

(c) (i) Write down the modal class.

(ii) Write down the mid-interval value for this class. [2]

(d) Use Elsie's data to calculate an estimate of the mean time that people spent in the library. [2]

(e) Using the table, write down the maximum possible number of people who spent 35 minutes or less in the library on that day. [1]

Elsie assumes her data to be representative of future visitors to the library.

(f) Find the probability a visitor spends at least 60 minutes in the library. [2]

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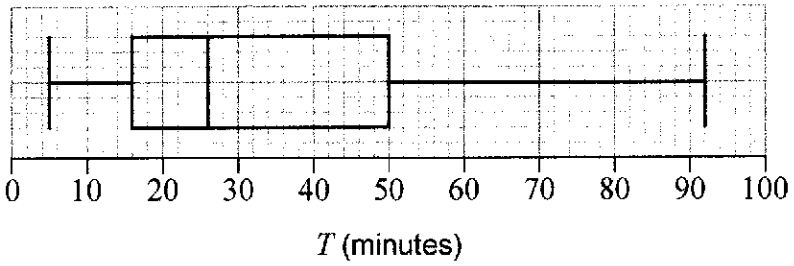
(Question 1 continued)

The following box and whisker diagram shows the times, in minutes, that the 160 visitors spent in the library.



(g) Write down the median time spent in the library. [1]

(h) Find the interquartile range. [2]

(i) Hence show that the longest time that a person spent in the library is not an outlier. [3]

Elsie believes the box and whisker diagram indicates that the times spent in the library are not normally distributed.

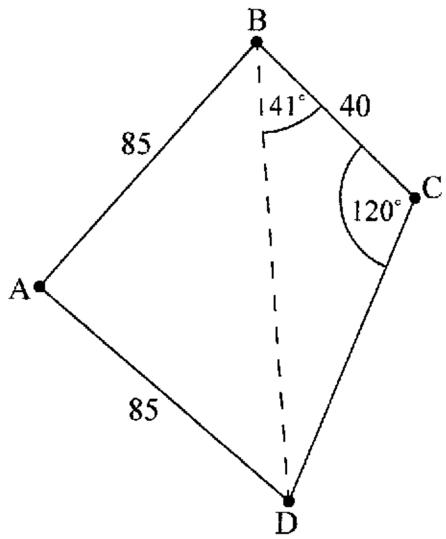
(j) Identify one feature of the box and whisker diagram which might support Elsie's belief. [1]

2. [Maximum mark: 17]

The following diagram shows a park bounded by a fence in the shape of a quadrilateral ABCD. A straight path crosses through the park from B to D.

$$AB = 85\,\text{m},\ AD = 85\,\text{m},\ BC = 40\,\text{m},\ C\hat{B}D = 41^\circ,\ B\hat{C}D = 120^\circ$$

diagram not to scale



(a) (i) Write down the value of angle BDC.

(ii) Hence use triangle BDC to find the length of path BD. [4]

(b) Calculate the size of angle $B\hat{A}D$, correct to five significant figures. [3]

The size of angle $B\hat{A}D$ rounds to 77°, correct to the nearest degree. Use $B\hat{A}D = 77^\circ$ for the rest of this question.

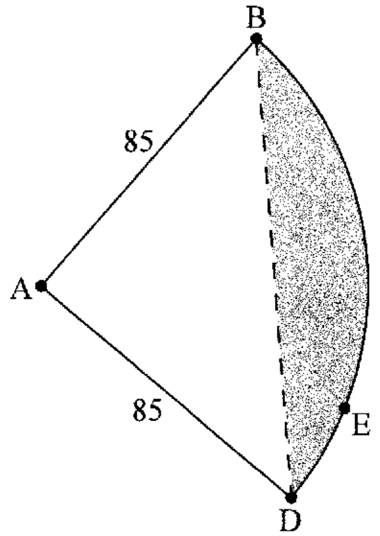
(c) Find the area bounded by the path BD, and fences AB and AD. [3]

(This question continues on the following page)

(Question 2 continued)

A landscaping firm proposes a new design for the park. Fences BC and CD are to be replaced by a fence in the shape of a circular arc BED with center A. This is illustrated in the following diagram.

diagram not to scale



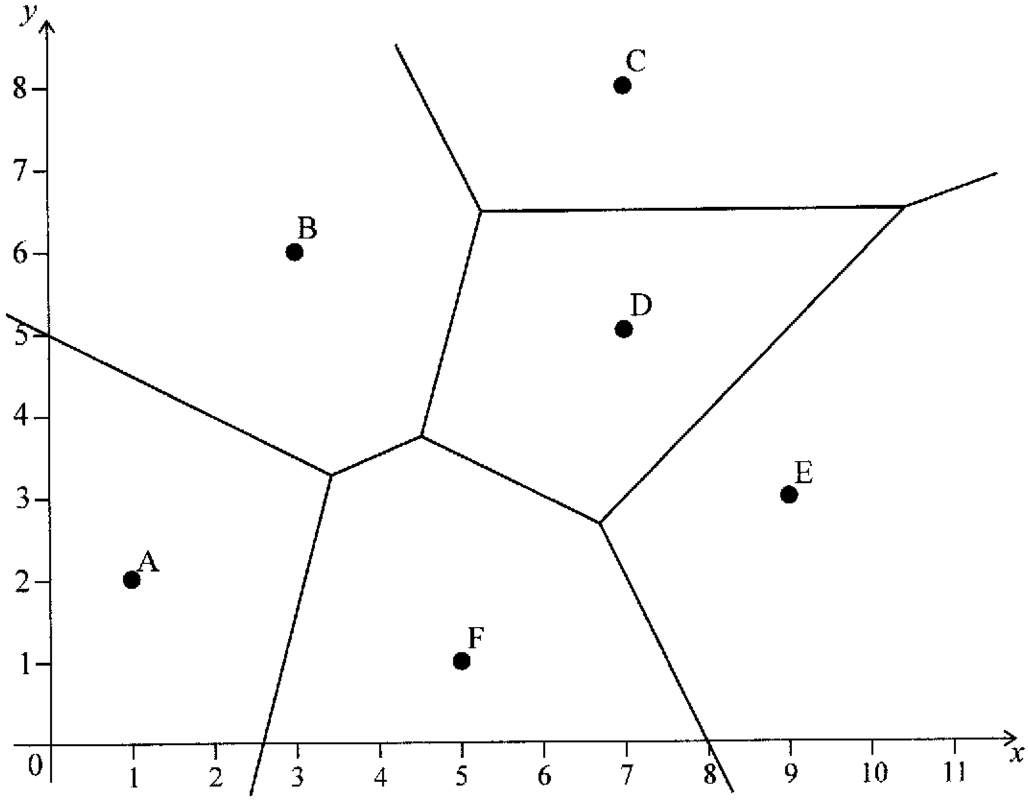
(d) Write down the distance from A to E. [1]

(e) Find the perimeter of the proposed park, ABED. [3]

(f) Find the area of the shaded region in the proposed park. [3]

3. [Maximum mark: 13]

Six restaurant locations (labelled A, B, C, D, E and F) are shown, together with their Voronoi diagram. All distances are measured in kilometres.



(a) Elena wants to eat at the closest restaurant to her. Write down the restaurant she should go to, if she is at

(i) $(2, 7)$.

(ii) $(0, 1)$, when restaurant A is closed. [2]

Restaurant C is at $(7, 8)$ and restaurant D is at $(7, 5)$.

(b) Find the equation of the perpendicular bisector of CD. [2]

Restaurant B is at $(3, 6)$.

(c) Find the equation of the perpendicular bisector of BC. [5]

(d) Hence find

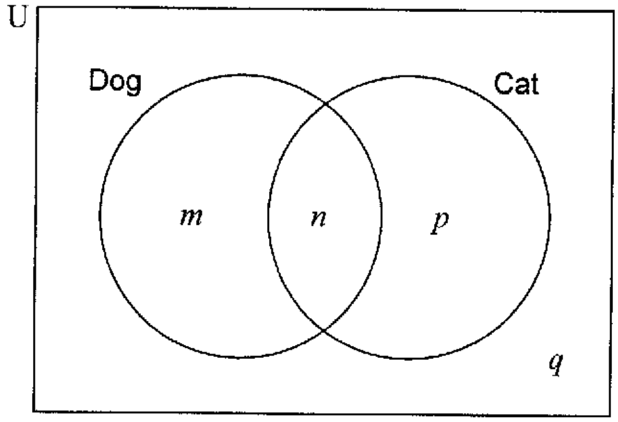
(i) the coordinates of the point which is of equal distance from B, C and D.

(ii) the distance of this point from D. [4]

4. [Maximum mark: 16]

At Mirabooka Primary School, a survey found that 68% of students have a dog and 36% of students have a cat. 14% of students have both a dog and a cat.

This information can be represented in the following Venn diagram, where m, n, p and q represent the percentage of students within each region.



(a) Find the value of

(i) m.

(ii) n.

(iii) p.

(iv) q. [4]

(b) Find the percentage of students who have a dog or a cat or both. [1]

(c) Find the probability that a randomly chosen student

(i) has a dog but does not have a cat.

(ii) has a dog given that they do not have a cat. [3]

(This question continues on the following page)

(Question 4 continued)

Each year, one student is chosen randomly to be the school captain of Mirabooka Primary School.

Tim is using a binomial distribution to make predictions about how many of the next 10 school captains will own a dog. He assumes that the percentages found in the survey will remain constant for future years and that the events "being a school captain" and "having a dog" are independent.

Use Tim's model to find the probability that in the next 10 years

(d) (i) 5 school captains have a dog.

(ii) more than 3 school captains have a dog.

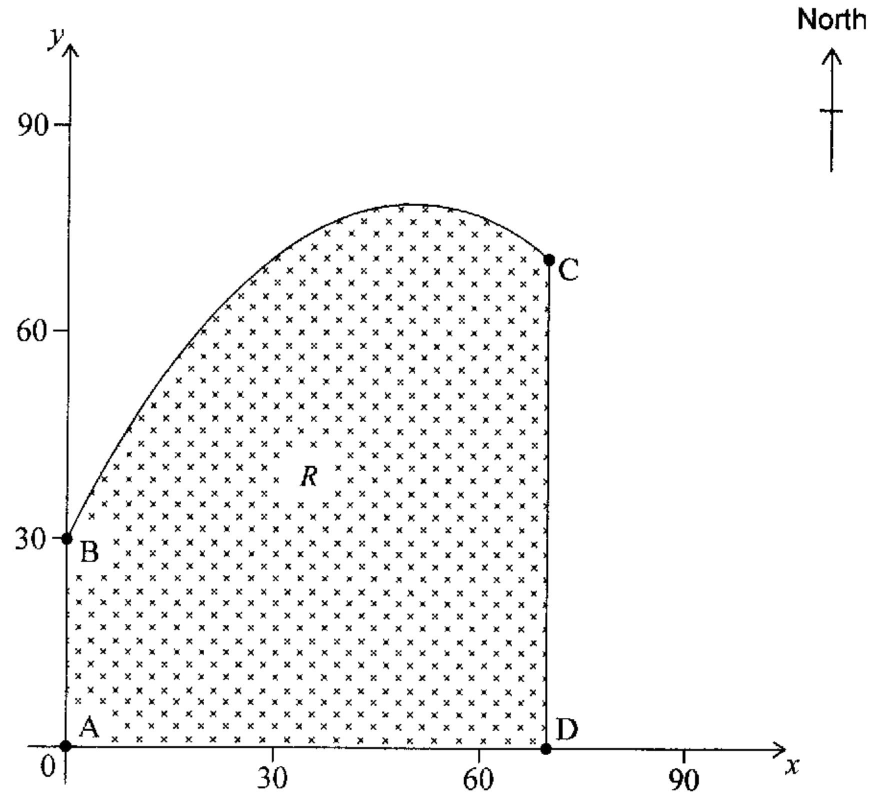
(iii) exactly 9 school captains in succession have a dog. [7]

John randomly chooses 10 students from the survey.

(e) State why John should not use the binomial distribution to find the probability that 5 of these students have a dog. [1]

5. [Maximum mark: 17]

Linda owns a field, represented by the shaded region R. The plan view of the field is shown in the following diagram, where both axes represent distance and are measured in metres.



The segments [AB], [CD] and [AD] respectively represent the western, eastern and southern boundaries of the field. The function, $f(x)$, models the northern boundary of the field between points B and C and is given by

$$f(x) = \frac{-x^2}{50} + 2x + 30, \text{ for } 0 \le x \le 70.$$

(a) (i) Find $f'(x)$.

(ii) Hence find the coordinates of the point on the field that is furthest north. [5]

Point A has coordinates (0, 0), point B has coordinates (0, 30), point C has coordinates (70, 72) and point D has coordinates (70, 0).

(b) (i) Write down the integral which can be used to find the area of the shaded region R.

(ii) Find the area of Linda's field. [4]

(This question continues on the following page)

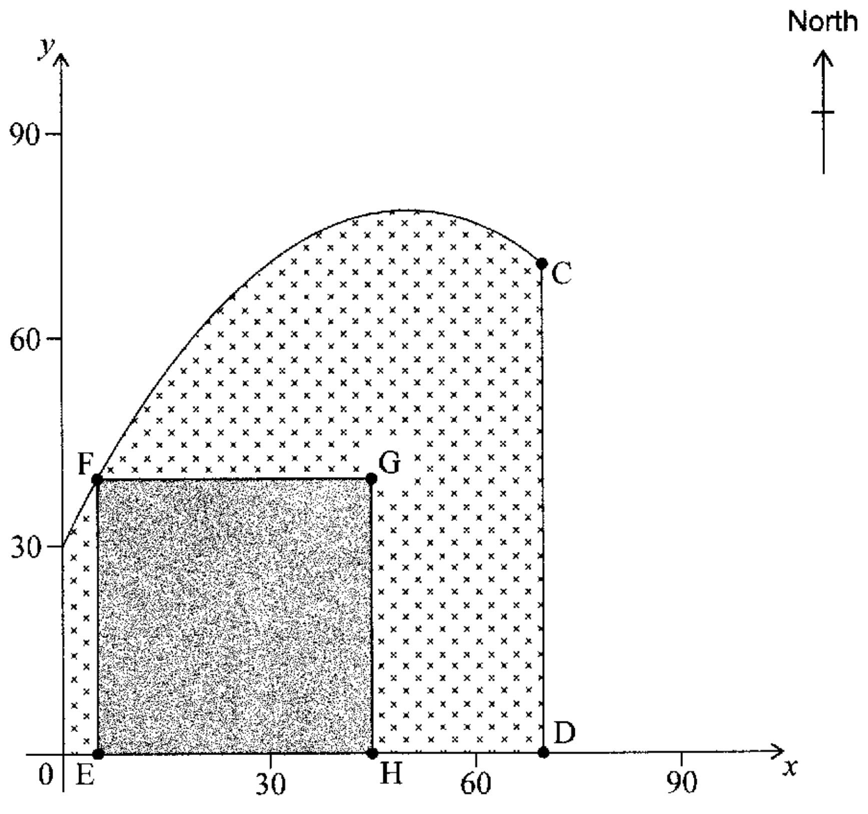
(Question 5 continued)

Linda used the trapezoidal rule with ten intervals to estimate the area. This calculation underestimated the area by $11.4\,m^2$.

(c) (i) Calculate the percentage error in Linda's estimate.

(ii) Suggest how Linda might be able to reduce the error whilst still using the trapezoidal rule. [3]

Linda would like to construct a building on her field. The **square** foundation of the building, EFGH, will be located such that [EH] is on the southern boundary and point F is on the northern boundary of the property. A possible location of the foundation of the building is shown in the following diagram.



The area of the square foundation will be largest when [GH] lies on [CD].

(d) (i) Find the x-coordinate of point E for the largest area of the square foundation of building EFGH.

(ii) Find the largest area of the foundation. [5]